HYDRODYNAMICS OF A TWO-PHASE STREAM IN THE DISPERSED ANNULAR FLOW REGIME

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UDC 532.529.5

Within the scope of a three-velocity, three-temperature, one-dimensional steady model we consider the hydrodynamics of dispersed annular, two-phase flows in cylindrical channels, characterized by the concurrent motion of an adhering liquid film and the core, or main body of the flow, which, in its turn, consists of a mixture of gas (vapor) and liquid droplets. It is assumed that each component of the mixture has its own velocity and temperature. Taken into account were phase transitions, the inhomogeneity of bulk velocities in the core and the film, and stripping of droplets from the surface of the film as well as precipitation onto it. For a description of the core of the flow, ideas developed in [1] were used. The equations obtained were applied in the determination of the pressure drop, the distribution of liquid between the film and the core of the flow, the slippage between phases, and the length of the stabilization interval. Concurrent motion of gas and liquid in a single channel with and without heat exchange is widely employed in power engineering and in the chemical industry. In such processes the dispersed annular regime of two-phase flow occurs over a wide range of variation in pressure and specific mass flow rates of the mixture at ratios of the mass flow rate of the vapor to the common flow rate of the mixture greater than 0.1-0.15, and this has been studied in many papers. General questions on the hydrodynamics of gas-liquid mixtures, applied in particular to a study of the motion of a two-phase flow in circular tubes, using hydrodynamic equations written for the mixture as a whole, were considered in [2]. In [3] the phenomenon of critical heat exchange in the flow of steam-water streams through tubes and in [4-6] hydraulic resistance were studied. Papers [7, 8] are devoted to the measurement of tangential stresses on a solid wall during flow of a two-phase current by the electrochemical method. Flow regimes of two-phase streams are considered in [9]. Mass exchange and slippage between phases are investigated in [10-13]. The character of the motion of a liquid film is studied in detail in [14]. Among existing models describing the flow of two-phase streams through tubes, the model in [15] should be pointed out. However, in this model, as in many others (for instance, Levi's model), the distinction between the liquid in the core of the flow (in the form of droplets) and the liquid in the film (which move with substantially different velocities) is not taken into account. This leads, on the one hand, to incorrect results on the velocities of the phases, and on the other, to the inapplicability of this model in the description of phenomena in which the liquid film is an important factor (for example, investigation of hydraulic resistance and critical heat exchange, which is related to the desiccation of an adhering liquid film).

1. Derivation of the Basic Equations. Suppose that a vapor-liquid mixture is flowing in a straight channel in the dispersed annular, stationary flow regime. We shall investigate the motion of such a medium under the usual assumptions made in the study of multiple-phase continuous media. We assume that in the core of the flow there is a second gas consisting of liquid droplets. The velocities and temperatures of the three components of the mixture (vapor, film, and droplets) can be different, and mass transfer between the phases is possible. There can be vaporization and condensation on the droplets and on the liquid film, and also stripping of droplets from the surface of the film and condensation onto it.

Everywhere in what follows parameters relating to the gas, the liquid film, and the droplets will be denoted by subscripts 1, 2, and 3, respectively.

$$\frac{dm_1}{dz} = J_{21} - J_{12} + J_{31} - J_{13}, \quad \frac{dm_2}{dz} = J_{12} - J_{21} + J_{32} - J_{23},$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 141-153, November-December, 1971. Original article submitted June 1, 1971.

Equations in differential form expressing conservation of mass for the vapor, the film, and the droplets are as follows:

$$\begin{gathered}\frac{dm_3}{dz} = J_{13} - J_{31} + J_{23} - J_{32} \\ \left(m_1 = \int\limits_{F_1} \rho_1 u_1 dF = \rho_1 F_1 u_1^m, \quad m_2 = \int\limits_{F_2} \rho_2 u_2 dF = \rho_2 F_2 u_2^m \right. \\ \left. m_3 = \int\limits_{F_1} \rho_3 u_3 dF = \rho_3 F_1 u_3^m, \quad \rho_1 = \rho_1^\circ \alpha, \quad \rho_2 = \rho_2^\circ, \quad \rho_3 = \rho_3^\circ (1-\alpha) \right)\end{gathered} \tag{1.1}$$

Here m_j ($j=1, 2, 3$) is the flux of the j-th component of the mixture through a perpendicular cross section of the channel; ρ_j, ρ_j^0, u_j^m are respectively the mean and the actual densities and the mean flow velocity of the j-th component of the mixture; α is the spatial volume of the gas (vapor) in the core of the flow; $F_1(z)$, $F_2(z)$ are the portions of the cross-sectional area of the channel that are occupied by the core of the flow and by the liquid film respectively, with $F_1(z) + F_2(z) = F$, where F is the area of the perpendicular cross section of the channel; and J_{kj}($k, j = 1, 2, 3$; $k \neq j$) are the intensities of the processes of mass transfer between components of the mixture, $J_{kj} \geq 0$ being the amount of matter of the k-th component changing into matter of the j-th component per unit time per unit length of the channel. Here, following [1], mass transfers between media are by convention divided into two reactions (each of which, from kinetic considerations, has a forward and a reverse reaction). The necessity for this division is related to the fact that the mass transfers k → j and j → k can lead to different changes in the momentum and energy of specific components of the mixture.

Adding Eqs. (1.1), we obtain the equation of conservation of mass for the whole system

$$\sum_{j=1}^{3} \frac{dm_j}{dz} = 0$$

Projected on the z axis, the equation of conservation of momentum for each medium separately is, account being taken of (1.1),

$$\begin{gathered} m_1 \frac{du_1^u}{dz} = -\alpha F_1 \frac{dp}{dz} - \rho_3 F_1 f - f_{12} + J_{21}(u_{21} - u_1^u) + J_{12}(u_1^u - u_{12}) + \\ + J_{13}(u_1^u - u_{13}) + J_{31}(u_{31} - u_1^u) - \rho_1 F_1 g_1 \\ m_2 \frac{du_2^u}{dz} = -F_2 \frac{dp}{dz} + f_{12} - f_w + J_{21}(u_2^u - u_{21}) + J_{12}(u_{12} - u_2^u) + \\ + J_{32}(u_{32} - u_2^u) + J_{23}(u_2^u - u_{23}) - \rho_2 F_2 g_2 \\ m_3 \frac{du_3^u}{dz} = -(1-\alpha) F_1 \frac{dp}{dz} + \rho_3 F_1 f + J_{13}(u_{13} - u_3^u) + J_{31}(u_3^u - u_{31}) + \\ + J_{23}(u_{23} - u_3^u) + J_{32}(u_3^u - u_{32}) - \rho_3 F_1 g_3 \\ \int\limits_{F_1} \rho_1 u_1^2 dF = m_1 u_1^u, \quad \int\limits_{F_2} \rho_2 u_2^2 dF = m_2 u_2^u, \quad \int\limits_{F_1} \rho_3 u_3^2 dF = m_3 u_3^u \end{gathered} \tag{1.2}$$

Here $u_j^{(u)}$ ($j=1, 2, 3$) is the mean momentum velocity of the j-th component of the medium; f is the interaction force between the gas and the droplets, referred to a drop as unit of mass, because the velocities of the phases in the core of flow do not coincide (friction, added masses, Magnus force, etc.); f_{12} is the frictional force on the surface separating the liquid film and the gas (this force can be equated to its axial component because the cross section through which the main body of the flow passes widens slowly, it being assumed that the thinning or thickening of the film along the length of the channel proceeds sufficiently slowly); and f_w is the frictional force between the liquid film and the solid wall of the channel. The terms $J_{kj}u_{kj}$ ($k, j = 1, 2, 3$; $k \neq j$) represent the respective momentum changes of the various components of the mixture due to mass transfer from the k-th to the j-th component, u_{kj} being the velocity of the k-th component at the separation boundary between the k-th and j-th components of the mixture. The last terms on the right-hand sides of Eqs. (1.2) are the projections of the body forces on the z axis.

Adding the appropriate equations of (1.2), we obtain the projection on the z axis of the equation of conservation of the momentum of the entire mixture

$$\frac{d}{dz} \sum_{j=1}^{3} m_j u_j^u = -F \frac{dp}{dz} - f_w - \sum_{j=1}^{3} \rho_j F_j g_j$$

In a manner similar to that used in connection with (1.2) one can obtain the equation for conservation of the energy of the entire mixture

$$\frac{d}{dz}\left\{\sum_{j=1}^{3} m_j \left[e_j + \frac{(u_j^e)^2}{2}\right]\right\} = -\frac{d}{dz}\left[p\left(\alpha F_1 u_1^m + F_2 u_2^m + (1-\alpha) F_1 u_3^m\right)\right] - \sum_{j=1}^{3} m_j g_j + q_w \tag{1.3}$$

$$\int_{F_j} e_j^\circ \rho_j u_j dF = m_j e_j, \qquad \int_{F_j} \rho_j u_j^3 dF = m_j (u^e)^2$$

Here e_j, e_j^0, u_j^e are, respectively, the mean and the actual internal energy and the energetic velocity of the j-th component of the mixture, and q_w is the external heat influx per unit time per unit length of the channel.

We write down equations for the internal energy of the film and the droplets, considering the droplets and the liquid film to be incompressible, $\rho_2^0 = \rho_3^0 = \text{const}$, since the work of pressure forces from the vapor on the deformation of the droplets and film is equal to zero. The surface energy of the liquid film and the droplets can also be neglected.

The internal energy of the film and the droplets will change on account of heat influx, heat exchange processes, and energy dissipation due to the action of interphase frictional forces. Moreover, the internal energy of the film changes on account of energy dissipation resulting from friction on the wall of the channel. Thus the equations for the internal energy of the film and the droplets have the form

$$\begin{aligned} \frac{d(m_2 e_2)}{dz} &= q_w - q_{21} + J_{12}e_{12} - J_{21}e_{21} + J_{32}e_{32} - J_{23}e_{23} + \\ &+ f_{12}(u_{12} - u_2^u) + f_w u_2^u + p(J_{12} - J_{21})\left(\frac{1}{\rho_1^\circ} - \frac{1}{\rho_2^\circ}\right) \\ \frac{d(m_3 e_3)}{dz} &= \rho_3 F_1 q_{13} + J_{13}e_{13} - J_{31}e_{31} + J_{23}e_{23} - J_{32}e_{32} + \\ &+ \rho_3 F_1 f(u_{13} - u_3^u) + p(J_{13} - J_{31})\left(\frac{1}{\rho_1^\circ} - \frac{1}{\rho_3^\circ}\right) \end{aligned} \tag{1.4}$$

Here q_{21} is the heat exchange between the film and the gas per unit time per unit length of the channel; q_{13} is the heat exchange between the gas and the droplets, referred to the unit of mass of the droplets; $J_{kj}e_{kj}$ ($k, j = 1, 2, 3$; $k \neq j$) is the change in internal energy of the film or the droplets due to mass transfer from the k-th to the j-th component of the mixture; and e_{kj} is the internal energy of material undergoing a transition from the k-th to the j-th component. The last terms in Eqs. (1.4) are equal to the work of pressure forces of the gas on the condensing and vaporizing matter.

Together with the equations of state, the assumptions regarding u_{kj}, e_{kj}, the relationship between u_j^m, u_j^u, u_j^e, and the relationships for f_{12}, f_w, f, J_{kj}, q_{21}, q_{13}, the system of differential equations (1.1)-(1.4) form a closed system in a region of continuous flow.

2. Relationships among Parameters on the Separation Boundary between Components of the Mixture and Averaged Characteristics of the Flow. For u_{31}, u_{13}, e_{12}, e_{13}, e_{21}, e_{31} it is natural to make assumptions similar to those in [1]:

$$u_{31} = u_{13} = u_3^u, \quad e_{12} = e_{13} = e_{2s}, \quad e_{21} = e_{31} = e_{1s}$$

Here the lower index s refers to the saturated state. As for u_{12}, u_{21}, u_{23}, u_{32}, e_{23}, e_{32}, we assume the following:

$$u_{12} = u_{21}, \; u_{23} = u_2', \; u_{32} = u_3^u, \; e_{32} = e_3$$

In order to obtain a relationship between u_j^m, u_j^u, u_j^e, u_2' and u_2^m, e_{23}, and e_2 it is necessary to prescribe velocity and temperature profiles in the film and in the core of the flow.

In the case of turbulent flow in the film [16] at Reynolds numbers $R_2 > 300$-400 ($R_2 = u_2^m \delta / \nu_2$, where δ is the thickness of the liquid film and ν_2 is the coefficient of kinematic viscosity of the liquid) and of turbulent flow in the core it can be assumed that the velocity profile conforms to the step law

$$\begin{aligned} \frac{u_k - u_2'}{u_k^\circ - u_2'} &= \left(1 - \frac{2r}{D - 2\delta}\right)^{n_k} \quad \left(k = 1, 3; \; 0 \leqslant r \leqslant \frac{D - 2\delta}{2}\right), \\ u_2 &= B\left(1 - \frac{2r}{D}\right)^{n_2} \quad \left(\frac{D - 2\delta}{2} \leqslant r \leqslant \frac{D}{2}\right) \end{aligned} \tag{2.1}$$

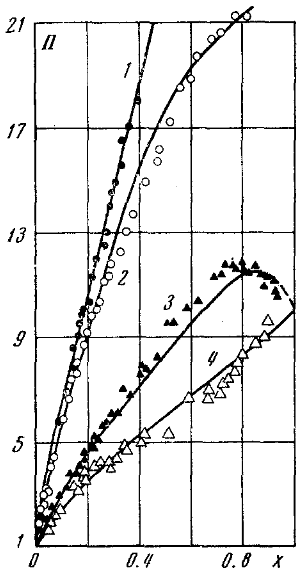

Fig. 1

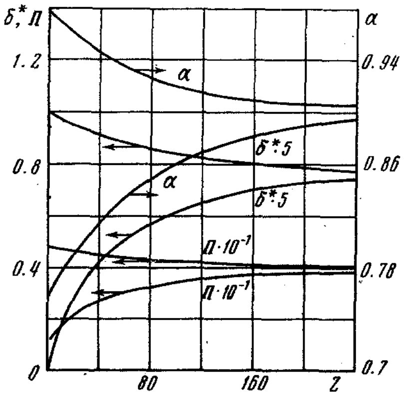

Fig. 2

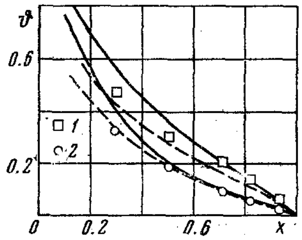

Fig. 3

where D is the diameter of the channel, B and u_k° are constants, while $n_j = {}^1/_9 - {}^1/_6$ (j = 1, 2, 3).

Then within an accuracy of $O(n_j^2)$ and $O(n_2 \delta/D)$

$$u_j^m = u_j^u = u_j^e = u_j \quad (j = 1, 2, 3), \quad u_2' = (1 + n_2) u_2^m \tag{2.2}$$

For laminar flow of the film ($R_2 < 300$–400)

$$u_2 = B\,(1 - 4r^2 / D^2)$$

In this case

$$u_2^u = {}^4/_3 u_2^m, \quad u_2' = 2u_2^m, \quad u_2^e = u_2^m \sqrt{2} \tag{2.3}$$

From (2.2) and (2.3) the unified notation

$$u_2^u = a_1 u_2^m, \quad u_2' = a_2 u_2^m, \quad u_2^e = a_3 u_2^m$$

follows.

3. Interaction between Components of the Mixture. In order to close the system of differential equations obtained above it is necessary to obtain relationships for the strong interaction between components of the mixture (f_{12}, f, f_w), thermal interaction (q_{21}, q_{13}, q_w), and the mass exchange between the media J_{kj} (k, j = 1, 2, 3; k ≠ j). All these processes are complicated and still require detailed study. For their determination it is necessary to know the flow regime and the constitution of the film (the structure of its surface, the relationship between its wave characteristics and the thickness δ and velocities of the phases, etc.), the constitution of the droplets (shape of the droplets, their dimensions, etc.). The frictional force between the gas and the film can be represented in the form

$$f_{12} = {}^1/_2\, C_{12}\pi\,(D - 2\delta)\,\rho_1^\circ\,(u_1 - u_2')^2, \quad C_{12} = C_{12}\,(\delta / D, R_1, R_2), \quad R_1 = (u_1 - u_2')\,(D - 2\delta) / \nu_1 \tag{3.1}$$

where ν_1 is the coefficient of kinematic viscosity of the vapor.

The friction between the gas (vapor) and the film is directly related to the flow regimes of the surface of the liquid film, which are determined by the velocities of the phases and the thickness of the film. By convention these regimes may be divided into three types: a wave type with large-scale waves, a ripple wave type, and a smooth-film regime. A thorough experimental investigation of the flow regimes of the film surface and of the demarcations between them is still needed. Direct measurements of the tangential stresses on the separation surface between the gas and the liquid film when the processes of stripping moisture from this surface and precipitation upon it are occurring do not seem possible at the present time. We remark that the measurement of the tangential stresses on the separation surface between the liquid film and the solid wall of the channel is possible, for instance, by the electrochemical method [7, 8].

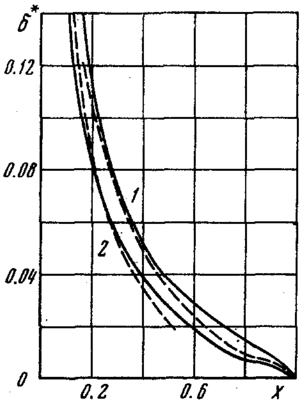

Fig. 4

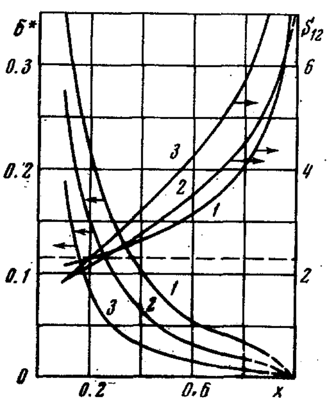

Fig. 5

The coefficient of friction depends principally on the nature of the damping of the waves in the same way as this would take place in rough tubes, since the core of the flow moves as if it were in a channel with liquid walls.* However, their "roughness" varies markedly over a wide range, depending on the flow regime of the film and of the core of the flow.

The coefficient of friction for flow of a single-phase fluid in a rough tube is approximated by [18]

$$C_1 = [2.28 - 4\lg(\varepsilon / D - 5y^* / D)]^{-2}, \qquad \varepsilon / y^* > 12$$

$$C_1 = 0.0008 + 0.0553 R_1^{-0,237}, \qquad \varepsilon / y^* < 5 \tag{3.2}$$

For $5 < \varepsilon / y^* < 12$ a linear variation between values given by the first and the second of the formulas can be taken. Here ε is the height of a protuberance in the sand-roughened pipe and

$$y^* = \frac{D}{R_1} \sqrt{\frac{2}{C_1}}$$

As a first approximation we assume that (3.2) is valid for C_{12}, the magnitude of the roughness being variable, and we take it to be equal to the thickness of the film [5]

$$\varepsilon = \eta\delta, \qquad \eta = \eta\,(\delta, R_2)$$

On the basis of an analysis of experimental data on hydraulic resistance in flows of gas-liquid mixtures through tubes [5] the mean value $\eta = 6$ can be obtained for the flow regime of a film with large-scale waves. In the flow regime of a film with ripples η falls to 0.6-1.0 [5, 6].

In the case of large Reynolds numbers for the film ($R_2 > 10^3$) the influence of R_2 on η can evidently be neglected and the dependence on δ can be represented as

$$\eta = N_1 - N_2 \exp[(N_3 - \delta) N_4] \qquad (N_1 \approx 6,\ N_2 \approx 5.5)$$

where N_3, N_4 are constants, determined more accurately through comparison with experimental data on hydraulic resistance.

When the liquid film becomes very thin there is the possibility of its disintegration [19]. Without going into details of the disintegration process at this stage, we assume that, when the film becomes thinner than a certain constant thickness δ°, it disintegrates and turns into a dry patch. According to the estimate of [19] $\delta^\circ = 20$-$50\ \mu$. In this case the vapor (gas) moves partly along a wet, and partly along a dry wall, and therefore by convention an expression for the frictional force is then constructed from a combination of (3.1) and $f_1 = C_1 \pi D \rho_1{}^\circ u_1^2 / 2$, for instance, one of the form

$$f_{12}' = f_{12} - (f_{12} - f_1)(\delta^\circ - \delta) / \delta^\circ$$

We represent the frictional force between the liquid film and the solid wall of the channel in the form

$$f_w = C_w \pi D \rho_2{}^\circ u_2{}^m / 2, \qquad C_w = C_w(R_2, \delta / D)$$

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*The assumption that processes occurring in the flow of a gas around individual waves on the surface of a film are similar to those that occur at protuberances of a rough surface was first made as far back as [17].

The coefficient of friction C_W depends on the flow regime of the liquid film. For a smooth tube in the case of the regime of developed turbulence in the flow of the liquid film $R_2 > 1000$ [20], one can obtain from (2.1), taking $n_2 = {}^1/_7$,

$$C_w = 0.0570 / R_2^{0.25} \tag{3.3}$$

For laminar flow of a liquid film ($R_2 < 400$)

$$C_w = 4 / R_2 \tag{3.4}$$

As a first approximation, a linear interpolation between (3.3) and (3.4) can be taken in the transition regime $400 < R_2 < 1000$.

Of the forces acting from the gas on the droplets the most significant is the frictional force under the conditions considered here, and this can be represented as

$$f = \frac{3}{4} \frac{\rho_1^\circ}{\rho_2^\circ} \frac{C_{13}}{d} \frac{(u_1 - u_3)^3}{|u_1 - u_3|} \qquad \left(C_{13} = C_{13}(R_{13}, \alpha), \ R_{13} = \frac{|u_1 - u_3| d}{\nu_1} \right)$$

where d is the diameter of the droplets.

As a first approximation it can be assumed that the spatial concentration of the droplets is sufficiently small that the influence of α on C_{13} can be neglected. For C_{13} the following functional relationship is recommended [21] over a wide range of the Reynolds number R_{13}:

$$C_{13} = 24 R_{13}^{-1} + 4 R_{13}^{-0.33}, \qquad 700 \geqslant R_{13} \geqslant 0$$

The force due to the effect of apparent additional masses can be neglected for $d \geq 10^{-6}$ m and $\rho_3^\circ / \rho_1^\circ > 10$ [1], and the projection of the Joukowski force onto the z axis vanishes.

The appearance of droplets in the core of the flow is due to the stripping of liquid from the crests of the large-scale waves [14]. Therefore the characteristic stripping rate is $(u_1 - u_2')$. Fractionization of the droplets is usually estimated by use of the Weber number W_3, characterizing the ratio of dynamic effects to the capillary pressure

$$W_3 = \frac{\rho_1^\circ (u_1 - u_2')^2 d}{\sigma_2}$$

Here σ_2 is the coefficient of surface tension of the liquid composing the droplets. In [22] considerations based on an approximate theory of elliptical deformation of the droplets due to an instantaneous application of the forces resulting from the flow around them yielded the characteristic number $W_3^* = 5.4$, corresponding to their disintegration. We assume that droplets for which the Weber number exceeds this characteristic value disintegrate into droplets of smaller diameter with the same total mass, the diameter now corresponding to this characteristic value of the Weber number.

To estimate the intensity of precipitation J_{32} we can use the empirical formula of [10]

$$J_{32} = 0.069 (D - 2\delta) \rho_1^\bullet (u_3 - u_2') \left(\frac{\rho_3}{\rho_1} \right)^{0.74} R_{32}^{-0.25}, \qquad R_{32} = \frac{(D - 2\delta)(u_3 - u_2')}{\nu_1} \tag{3.5}$$

Expression (3.5) is written in terms of the variables used in this paper, the characteristic velocity being taken as $u_3 - u_2'$, and not u_1. (In [10] the precipitation of droplets on an immovable wall was studied.) It should be remarked that, applying this empirical relationship for the intensity of precipitation, into which the diameter of the droplets does not enter, and keeping in mind that in the core of the flow the droplets move with a velocity close to that of the ambient medium, one can conclude that the diameter of the droplets has little influence on the solution of the system.

The process of stripping of droplets from the surface of the liquid film has been studied in many papers, in particular [5, 11, 14]. In most of this work the intensity of stripping of the liquid equalizes that of the precipitation, though this is true only in the case of equilibrium. We define the equilibrium case as a state of the system in which the velocities of the components of the mixture along the tube do not vary (for flow of the mixture in an unheated channel). It has been shown [11] that for comparatively thick films the gas velocity at which stripping begins depends only on the surface tension σ_2 of the liquid and not on its viscosity. Therefore, in this case, it can be assumed that the transition from a flow regime of the film without stripping of droplets to one with stripping is determined by equality of the dynamic pressure of the liquid at some point of a wave crest and the capillary pressure at the same point:

$$h\rho_2^{\circ}(u_2')^2 / 2 = \sigma_2 / \delta$$

where h is some constant factor. Consequently the condition for the onset of stripping is determined by the Weber number W_2

$$W_2 = \frac{\rho_2^{\circ}(u_2')^2\delta}{\sigma_2}$$

. And the stripping of droplets will obviously proceed with greater intensity, the greater the divergence of the value of W_2 from some critical value $W_2{}^*$.

In first approximation we can employ a linear relationship connecting the intensity of the stripping with the divergence of W_2 from $W_2{}^*$,

$$\frac{J_{23}}{\pi(D-2\delta)\rho_1^{\circ}(u_1-u_2')} = A(W_2 - W_2{}^*) \quad (W_2 > W_2{}^*), \quad J_{23} = 0 \quad (W_2 \leqslant W_2{}^*)$$

At the same time, analysis of stripping phenomena with invocation of dimension theory suggests the following expression for the proportionality coefficient A:

$$A = \varkappa(\rho_2^{\circ} / \rho_1^{\circ}) \quad [(u_1 - u_2') / u_2']^m$$

where $\varkappa$, n, m are constants. Together with $W_2{}^*$ these constants can be estimated, for example, from experimental data on critical heat exchange of the second kind in a heated tube, this being related to the desiccation of the adhering liquid film. From these data [3] one determines values of the mass flow rate vapor content $x_{bn}{}^{\circ}$ (i.e., the ratio of the mass flow rate of the vapor to the common mass flow rate of the mixture in the channel) at which complete desiccation of the liquid film occurs, and also values of the mass flow rate vapor content $x_{\Delta p}$ at which a change in flow regimes of the liquid film occurs: from a regime with stripping of liquid from its surface to a regime where this stripping ceases. Knowing the increment in vapor content due to vaporization of the liquid film, one can estimate the flow rate in the film both at the entrance to the heated portion of the experimental region and at the moment when the regimes change. It has been shown experimentally that for relatively high heat fluxes q_w precipitation of droplets on the film does not occur because they are blown off by the vapor that is evaporating from the film.

The constants $\varkappa$, n, and m can be determined from data on the flow rate of liquid in the film. By use of certain considerations [13] concerning the relative slippage $u_1/u_2{}^m$ the value of $W_2{}^*$ was estimated, which was found to be in the range 20-50. Reference [4] gives results of a systematic study of the hydraulic resistance in flow of a vapor–water mixture in tubes with and without an external heat supply, the pressures at the entrance having a range of p = 49-196 bar, and the specific mass flow rates of the mixture being in the range w = 500-2000 kg/m² · sec. In the regimes indicated above a two-phase mixture can be considered within the scope of a three-velocity, but still a single-temperature, model, where the temperature of the mixture in every cross section of the channel is equal to the saturation temperature T_S for the pressure at that cross section. Neglecting the variation in internal energy of the film and that of the droplets with distance along the channel (because of the smallness of the ratio $\Delta p/p$, where Δp is the pressure drop over some characteristic distance) and the variation in the internal energy of the film due to the work of frictional forces, we obtain from (1.4)

$$J_{21} = q_w / r, \quad J_{12} = J_{13} = J_{31} = 0$$

For the components of the mixture we take the simplest equations of state, namely the equation of state of an ideal gas for the vapor (gas) and the condition of incompressibility for the liquid.

4. Equations for the Solution of the Problem. Thanks to the simplifications made above, we obtain a system of six differential equations of motion together with a known relationship for $T_S = f(p)$. The Cauchy problem for this system is to be solved in the case of an ascending, dispersed, annular, two-phase flow in a vertical channel.

We convert to dimensionless variables

$$M_j = \frac{4m_j}{\rho_1^{\circ}u_{10}\pi D}, \quad U_j = \frac{u_j}{u_{10}} \quad (j = 1, 2, 3), \quad \delta^* = \frac{2\delta}{D}, \quad Z = \frac{2z}{\pi D},$$

$$J_{21}{}^* = \frac{2J_{21}}{\rho_1^{\circ}u_{10}D}, \quad J_{23}{}^* = \frac{2J_{23}}{\rho_1^{\circ}u_{10}D}, \quad J_{32}{}^* = \frac{2J_{32}}{\rho_1^{\circ}u_{10}D}, \quad f^* = \frac{f\pi D}{2u_{10}{}^2}$$

$$f_{12}{}^* = \frac{2f_{12}}{\rho_1^{\circ}u_{10}{}^2D}, \quad f_w{}^* = \frac{2f_w}{\rho_1^{\circ}u_{10}{}^2D}, \quad P = \frac{p}{\rho_2^{\circ}u_{10}{}^2}$$

$$\left(\frac{4m_1}{\pi D^2} = \rho_1^\circ \alpha (1 - \delta^*)^2 u_1, \ \frac{4m_2}{\pi D^2} = \rho_2^\circ (2\delta^* - \delta^{*2}) u_2, \ \frac{4m_3}{\pi D^2} = \rho_2^\circ (1 - \alpha)(1 - \delta^*)^2 u_3\right)$$

Here quantities distinguished by the subscript 0 correspond to initial data at Z = 0. The equation of state of the vapor has the form

$$\rho_1^\circ = \rho_{10}^\circ \ PT_0 / P_0 T$$

We write the system of equations of motion in dimensionless form:

$$\begin{gathered}
\alpha (1 - \delta^*)^2 \frac{dU_1}{dZ} - 2U_1 \alpha (1 - \delta^*) \frac{d\delta^*}{dZ} + U_1 (1 - \delta^*)^2 \frac{d\alpha}{dZ} = J_{21}^* \\
(2\delta^* - \delta^{*2}) \frac{dU_2}{dZ} + 2U_2 (1 - \delta^*) \frac{d\delta^*}{dZ} = \frac{J_{32}^* - J_{21}^* - J_{23}^*}{k} \\
(1 - \alpha)(1 - \delta^*)^2 \frac{dU_3}{dZ} - 2(1 - \alpha)(1 - \delta^*) U_3 \frac{d\delta^*}{dZ} - \\
- (1 - \delta^*)^2 U_3 \frac{d\alpha}{dZ} = \frac{J_{23}^* - J_{32}^*}{k} \\
\alpha (1 - \delta^*)^2 U_1 \frac{dU_1}{dZ} + k\alpha (1 - \delta^*)^2 \frac{dP}{dZ} = b_1 \\
a_1 (2\delta^* - \delta^{*2}) U_2 \frac{dU_2}{dZ} + (2\delta^* - \delta^{*2}) \frac{dP}{dZ} = \frac{b_2}{k} \\
(1 - \alpha)(1 - \delta^*)^2 U_3 \frac{dU_3}{dZ} + (1 - \alpha)(1 - \delta^*)^2 \frac{dP}{dZ} = \frac{b_3}{k}
\end{gathered} \tag{4.1}$$

Here

$$\begin{gathered}
b_1 = -k (1 - \alpha)(1 - \delta^*)^2 f^* - f_{12}^* + J_{21}^* (U_2' - U_1) - L\alpha (1 - \delta^*)^2 \\
b_2 = f_{12}^* - f_{w}^* - J_{21}^* (U_2' - a_1 U_2) - J_{23}^* (U_2' - a_1 U_2) + \\
+ J_{32}^* (U_3 - a_1 U_2) - kL (2\delta^* - \delta^{*2}) \\
b_3 = k (1 - \alpha)(1 - \delta^*)^2 f^* + J_{23} (U_2' - U_3) - kL (1 - \alpha)(1 - \delta^*)^2 \\
k = \rho_2^\circ / \rho_1^\circ, \qquad L = \pi D g / 2u_{10}^2, \qquad g = 9.81 \text{ m/sec}^2
\end{gathered}$$

The system (4.1) has one first integral for mass

$$M_1 + M_2 + M_3 = S = \text{const}$$

from which we obtain

$$\alpha = \frac{S - k [U_2 (2\delta^* - \delta^{*2}) + U_3 (1 - \delta^*)^2]}{(U_1 - kU_3)(1 - \delta^*)^2}$$

Solving (4.1) for the derivatives, we obtain

$$\begin{gathered}
\frac{dU_1}{dZ} = \frac{G_1}{G_4 \alpha (1 - \delta^*)^2}, \quad \frac{dU_2}{dZ} = \frac{G_2}{G_4 k (2\delta^* - \delta^{*2})}, \quad \frac{dU_3}{dZ} = \frac{G_3}{G_4 k (1 - \alpha)(1 - \delta^*)^2} \\
\frac{d\delta^*}{dZ} = \frac{1}{2U_2 (1 - \delta^*) k} \left[J_{32}^* - J_{21}^* - J_{23}^* - \frac{G_2}{G_4}\right] \\
\frac{dP}{dZ} = \frac{1}{G_1} [a_1 k b_1 U_2^2 U_3^2 + b_2 U_1^2 U_3^2 + a_1 b_3 U_1^2 U_2^2 - a_1 U_1 U_2 U_3 b_4] \\
G_1 = U_1 \{b_1 [(2\delta^* - \delta^{*2}) U_3^2 + (1 - \alpha)(1 - \delta^*)^2 a_1 U_2^2] - \\
- \alpha (1 - \delta^*)^2 [b_2 U_3^2 + a_1 b_3 U_2^2]\} + a_1 \alpha (1 - \delta^*)^2 U_2 U_3 b_4 \\
G_2 = U_2 \{b_2 [k\alpha (1 - \delta^*)^2 U_3^2 + (1 - \alpha)(1 - \delta^*)^2 U_1^2] - \\
- (2\delta^* - \delta^{*2}) [b_3 U_1^2 + k b_1 U_3^2]\} + (2\delta^* - \delta^{*2}) U_1 U_3 b_4 \\
G_3 = U_3 \{b_3 [(2\delta^* - \delta^{*2}) U_1^2 + a_1 k\alpha (1 - \delta^*)^2 U_2^2] - (1 - \alpha)(1 - \delta^*)^2 [k a_1 b_1 U_2^2 + \\
+ b_2 U_1^2]\} + a_1 (1 - \alpha)(1 - \delta^*)^2 U_1 U_2 b_4 \\
G_4 = (2\delta^* - \delta^{*2}) U_1^2 U_3^2 + a_1 (1 - \delta^*)^2 [k\alpha U_3^2 + (1 - \alpha) U_1^2] \\
b_4 = k J_{21}^* U_2 U_3 + (J_{32}^* - J_{21}^* - J_{23}^*) U_1 U_3 + (J_{23}^* - J_{32}^*) U_1 U_2
\end{gathered} \tag{4.2}$$

The system (4.2) was integrated numerically with fixed values of P_0, w, and x at the entrance of the channel under various initial conditions in order to study their influence on the equilibrium distance, i.e., the distance at which a state of the system is established where the velocities of the components of the mixture do not vary along the channel. The calculations were carried out for flows of the mixture in an unheated channel. Data on the thermodynamic properties of water and water vapor were taken from [23]. Calculation of the problem was performed up to the cross section where an equilibrium state of the system was established. The pressure gradient at this cross section was compared with the pressure drop per unit distance along the channel obtained in [4], leaving body forces out of account. By matching results with the experimental data for $p=49$ bar, $w=2000$ kg/m^2 · sec, $p=98$ bar, $w=2000$ kg/m^2 · sec, and $D=8$ mm, a refinement of the values of N_3 and N_4 was obtained ($N_3=60\ \mu$, $N_4=4\cdot 10^{-3}\ \mu^{-1}$).

From a comparison with experimental data on the critical heat exchange for the same regimes the following values were obtained:

$$\varkappa = 0.1\cdot 10^{-3},\ n = -1.0,\ m = -0.25$$

The film thickness at which disintegration begins was taken to be $\delta^\circ=50\ \mu$. Values of the constants obtained here cannot be considered to be final. Their further refinement requires a comparison of numerical and experimental results for various characteristics of a dispersed, annular, two-phase flow, in particular with a direct measurement of the flow rate of the liquid in the film over a wide range of variation in the regime parameters. Regrettably, data on the flow rate of liquid in a film are very meager at the present time.

5. A Few Results. Figures 1-5 show a few results of the numerical integration that illustrate the influence of p, w, and x on various flow characteristics of a dispersed, annular, two-phase flow in an unheated channel.*

Figure 1 shows results of calculations and experiments [4] on the hydraulic resistance for vapor–water flows in a smooth vertical tube with diameter $D=8$ mm. The specific mass flow rate x of the vapor content is plotted along the horizontal axis and the relative pressure drop Π is plotted along the vertical axis, the latter being equal to the ratio of the pressure drop in a two-phase flow due to friction and mass exchange between phases, disregarding pressure variations resulting from the action of body forces, to the pressure drop due to friction in a flow of water at the saturation temperature

$$\Pi = \left[\frac{dp}{dz}\right]_f \Big/ \left[\frac{dp}{dz}\right]_0$$

Curves 1, 2, 3, and 4 correspond to pressures of 49, 49, 98, and 98 bar, curves 1 and 2 being calculated for $w=1000$ kg/m^2 · sec and the others for $w=2000$ kg/m^2 · sec. Thus it is evident that, within the limits of the model under consideration, calculation of the influence of w on Π is a success.

For $p=98$ bar, $w=1000$ kg/m^2 · sec, $x=0.2$, and $D=8$ mm, Fig. 2 shows an example of the variation in the characteristics of dispersed annular flow as a function of distance from the entrance to the channel and various initial conditions.

We note that the distance at which the equilibrium state is established (this being at the cross section where there is equilibrium, $J_{32}\approx J_{23}$) depends strongly on the velocities of the components of the mixture. For comparatively low values of w and x this distance is larger than it is for large w and x. This agrees completely with the data of [11]. The equilibrium distance decreases with an increase in pressure.

In Fig. 3 values of the relative flow rate of liquid in the film $\nu=m_1/(m_1+m_2+m_3)$ obtained numerically are compared with the experimental data of [12], where the flow rate of liquid in the film was measured as a function of w and x at $p=69$ bar and $D=12.5$ mm at a distance 200 D from the entrance to the tube (points 1 correspond to $w=544$ kg/m^2 · sec, and points 2 to $w=950$ kg/m^2 · sec). In these experiments conditions at the entrance were such that at the initial cross section most of the liquid was in the core of the flow. The solid curves refer to the equilibrium cross section and the dashed curves refer to the cross section of the channel where the measurements were made, that is, at a distance of 200 D from the entrance. The calculations were carried out for the case where all the liquid was in the core of the flow at the initial cross section. From Fig. 3 it is clear that results of the calculations agree fairly well with the experimental data.

Figure 4 exhibits the dependence of δ^* on x. The dashed curves 1 and 2 interpolate the experimental data of [11] for a two-phase, argon–water flow in a vertical tube with $D=25$ mm, $p=21.4$ bar, at room temperature, for values $w=1000$ and 2000 kg/m^2 · sec, respectively. The solid curves display this dependence, obtained numerically under the same conditions, at a distance of 5 m from the entrance. From the experi-

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*A separate paper by the author will be devoted to an investigation of characteristics of flow of a dispersed annular stream in a heated channel.

mental arrangement in [11] it is difficult to estimate the conditions at the entrance. Calculations were carried out for the case where all the liquid was in the core of the flow at the initial cross section. Nevertheless it can be seen that there is good qualitative agreement between results of the calculations and the experimental data.

Figure 5 shows the influence of w and x on δ^* and on the slippage $S_{12} = u_1/u_2{}^m$ for $p = 98$ bar and $D = 8$ mm. Here the curves 1, 2, and 3 for δ^* and S_{12} refer respectively to $w = 500$, 1000, and 2000 kg/m² · sec. The dashed horizontal line corresponds to the value $S_{12} = (\rho_2^\circ/\rho_1^\circ)^{1/3}$ for annular, two-phase flows proposed in [13]. Obviously such a simple dependence cannot approximate the data on slippage, which depends not only on the pressure, but on x and w as well.

Thus, for many characteristics of dispersed annular flow fairly satisfactory agreement with the experimental data of various authors has been obtained, notwithstanding the approximate character of certain estimates. Of course, this agreement could, to some degree, be a result of mutual influence of the assumptions that were made. But at the same time the approach we have developed also enables us to estimate the effect of various processes and interactions on the general picture of the flow in a dispersed annular stream. For instance, as a result of the numerical integration of the system (4.2), we can note that the constants entering into relationships for the force of friction f_{12} and the intensity of stripping from the film surface J_{23} influence the flow characteristics fairly independently of one another. The first influences the pressure gradient strongly, while the second affects the flow rate and thickness of the liquid film. An investigation of the influence of the value of the critical number $W_2{}^*$, which characterizes the onset of removal of moisture from the film surface, was carried out. It was found that a decrease in $W_2{}^*$ from 40 to 15 has little effect on characteristics of a dispersed annular flow. We also remark that our constructive procedure makes it possible, on the basis of quantities that are easily measured experimentally (w, Π, J_{32}, x), to obtain parameters and relationships for various processes in two-phase, dispersed, annular flows that would be difficult to determine experimentally, especially at high pressures and temperatures (u_j, m_j ($j = 1, 2, 3$), δ, α, J_{23}, equilibrium distance, etc.).

In conclusion the author thanks V. E. Doroshuka and S. S. Kutateladze for useful discussions and valuable comments.

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